Prescriptive Trees for Integrated Forecasting and Optimization Applied in Trading of Renewable Energy

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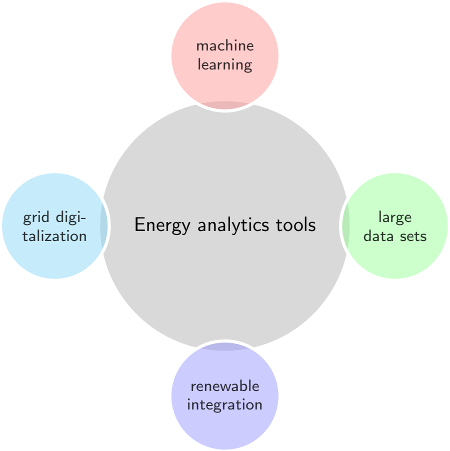
Context and Motivation

Data-driven decisions in two steps:

- *Forecast*: estimation of uncertain parameters (renewable production).
- *Optimize*: Derive an optimal set of actions (prescriptions).

Issues:

- Forecast accuracy $\neq$ forecast *value*
- Each parameter requires a separate forecasting model
- Impact of data on decisions is obscure



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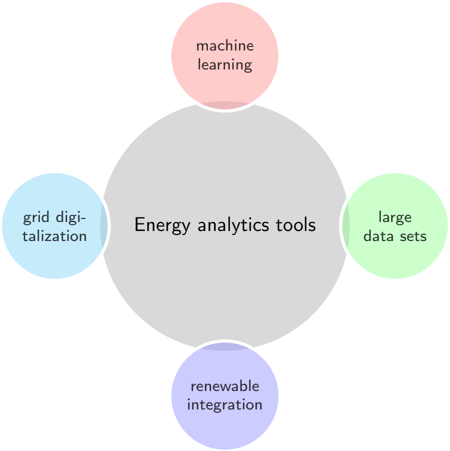

Smart4RES

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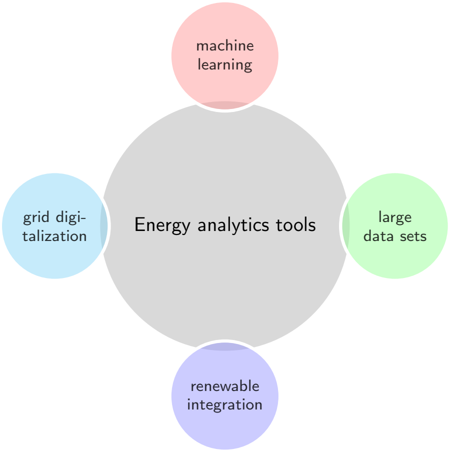
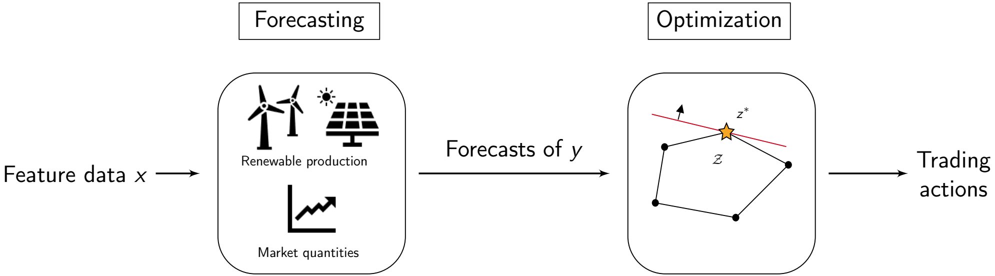


Closing the loop between forecasting and optimization to improve prescriptive performance in renewable trading applications.



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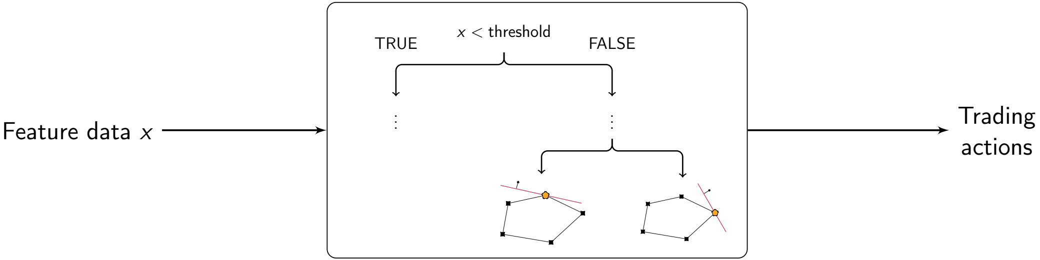
Integrated Forecasting-Optimization



Outline

Stochastic optimization problems with contextual information (or *prescriptive analytics problem*):

- Uncertain parameters Y: renewable production, market quantities
- Associated features X: weather forecasts, historical market data.
- Vector of decisions z: energy offers.

The goal is to minimize:

$$v = \min_{z \in \mathcal{Z}} \mathbb{E}_{\mathbb{Q}}[c(z;Y)|X = \overline{x}] = \min_{z \in \mathcal{Z}} \mathbb{E}_{y \sim \mathbb{Q}_{\overline{x}}}[c(z;Y)] \quad (1)$$

where $\mathcal{Z}$ the feasible set, $c(\cdot)$ the cost function, $\mathbb{Q}$ the joint distribution of (X, Y), $\overline{x}$ a new observation of X, and $\mathbb{Q}_{\overline{x}}$ predictive density of Y conditioned on $\overline{x}$.
Types of problems: constrained, multi-temporal, *single-stage*.

How to solve this problem

Training data set: $\{(y_i, x_i)\}_{i=1}^{n}$

Forecast, then optimize (FO): train forecasting model $f : \mathcal{X} \to \mathcal{Y}$, infer predictive densities (conditional expectation), solve stochastic (deterministic) problem.

Predictive Prescriptions: find similar observations, solve a weighted Sample Average Approximation (SAA) conditioned on $\overline{x}$ [HPB10, BK20]

$$\hat{z}(\overline{x}) = \arg\min_{z \in \mathcal{Z}} \sum_{i=1}^{n} \omega_{n,i}(\overline{x}) c(z; y_i), \tag{2}$$

$\omega_{n,i}(\overline{x})$: weights from local learning algorithms, e.g., kNN and decision trees.

- If $\omega_{n,i}(\overline{x})$ are derived by training for prediction: equivalent to **FO** with probabilistic forecasts.
- Proposed: derive $\omega_{n,i}(\overline{x})$ by directly **minimizing downstream costs**

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Search over functions $f : \mathcal{X} \to \mathcal{Y}$ to minimize in-sample decision costs using a weighted SAA of the original problem. Formally:

$$\min_{f \in \mathcal{F}, z^f(x_i) \in \mathcal{Z}} \quad \sum_{i \in [n]} c(z^f(x_i); y_i) \tag{3a}$$

$$\text{s.t.}$$

$$z^f(x_i) = \arg\min_{z \in \mathcal{Z}} \sum_{j \in [n]} \omega^f_{n,j}(x_i) c(z; y_j) \quad \forall i \in [n], \tag{3b}$$

where $[n] := \{1, \dots, n\}$.

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Prescriptive trees: trees that output prescriptions rather than predictions. Following CART [BFSO84], apply recursive binary splits:

$$\min_{j,s}\left[\min_{z_1\in\mathcal{Z}}\sum_{i\in R_1} c(z_1;y_i)+\min_{z_2\in\mathcal{Z}}\sum_{i\in R_2} c(z_2;y_i)\right]. \tag{4}$$

Inner min problems in (4) correspond to the SAA solution of each partition.

No analytical solution for constrained problems → Training is too costly!

Train ensemble with random splits [GEW06] to reduce costs:

1. At each node of each tree, sample a subset of K features from X.
2. For each feature, sample a candidate split point.
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Explainability: Measuring prescriptiveness

Q: What is the impact of features on prescriptive performance?

A: Feature importance for predictive accuracy $\rightarrow$ adapt to measure *prescriptiveness*.

- *Mean Decrease Impurity* (MDI): For each feature, measure the expected cost reduction when it is used at node splits (in-sample, no computational cost).
- *Permutation Importance*: Shuffle feature observations, derive prescriptions, find expected cost increase (out-of-sample, high computational cost).

Other ideas: Shapley value, LIME (for single prescription), etc.

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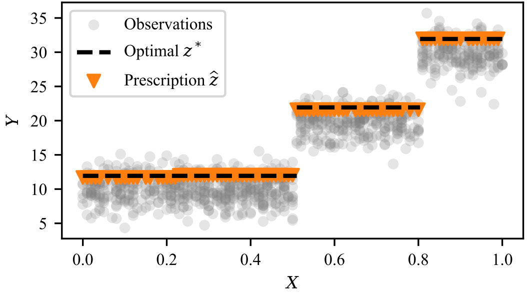
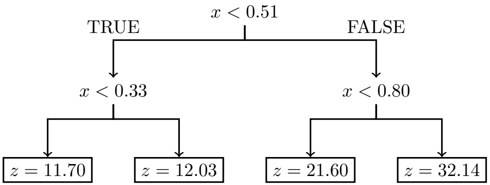
Toy newsvendor problem: cost function $c(z; Y) = 2(Y - z)^{-} + 10(z - Y)^{+}$, uncertain demand $Y = g(X) + \epsilon$, X single feature, ϵ noise.



In-sample fit



Tree structure

Outline

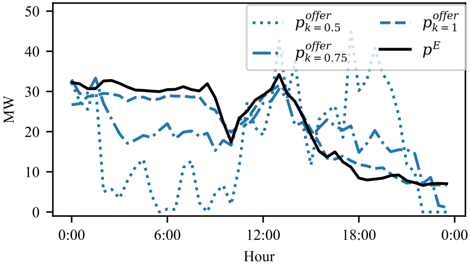
- **RES trading**: trading as a price-taker in a DA market, under single-/dual-pricing balancing mechanism.
- **RES trading with storage**: extend the above to include storage, co-optimize DA offers *and* storage control policy [SCMK22].
- **Clearing DA market**: stochastic market clearing with network constraints, test on IEEE-24 system

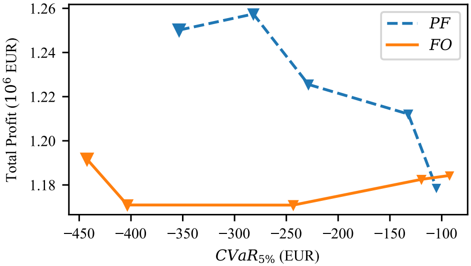
Objective: Balance trading performance (*prescriptive*) and forecast accuracy (*predictive*), jointly consider uncertainty in *both* energy and regulation penalties.

$$\min_{p^{offer}} \quad \mathbb{E}\left[\underbrace{(1-k)(-\rho^{\text{single/dual}})}_{\text{prescriptive}} + \underbrace{k\left\|p^{E}-p^{offer}\right\|_2^2}_{\text{predictive}}\right]$$
$$\text{s.t.} \quad p^{min} \leq p^{offer} \leq p^{max},$$

where $\rho^{\text{single/dual}}$ the profit function, k a design parameter controls the trade-off. For $k=0$ retrieve "0-1" or newsvendor (depends on market design) loss, for $k=1$ retrieve standard regression loss.

Illustrative results for a market with a single-price balancing mechanism:





Energy offers become riskier as k decreases.

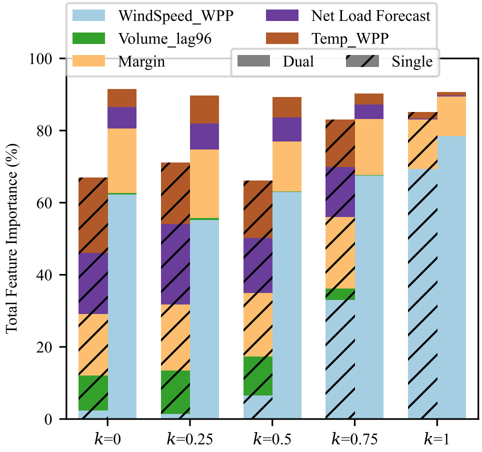
Risk-reward trade-off against the standard FO.

For $k = 0$, derive optimal trading offer:

- *Single*-price: offer is either 0 or 1, *only regulation costs* matter.
- *Dual*-price: offer is the optimal energy *quantile* given expected regulation costs.

For $k = 1$:

- Standard regression, offer expected production.



Norm. prescriptive feature importance.

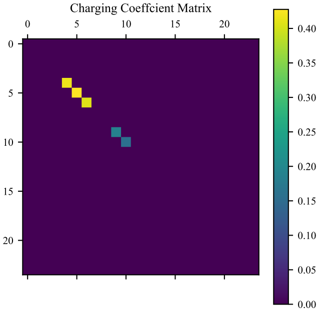
Objective: Trade-off between DA arbitrage actions and minimizing imbalance volume during real-time operation.
Linear decision rules for recourse actions: $\widetilde{p} = \widehat{p} + D\xi$, where $\widehat{p}$ the scheduled DA decisions, D lower-triangular coefficient matrix, ξ uncertainty (forecast error).

$$\min_{\mathcal{P}} \quad \mathbb{E}\left[\sum_{t=1}^{T} \underbrace{-(1-k)\pi_t^{da} p_t^{offer}}_{\text{DA arbitrage}} + \underbrace{k \left\| p_t^{output} - p_t^{offer} \right\|_2^2}_{\text{real-time control}}\right] \quad (5a)$$

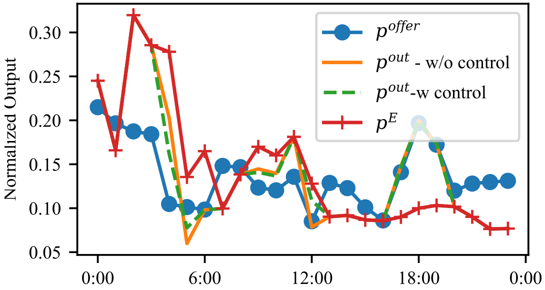
$$\text{s.t.} \quad \text{Offer limits, state transition function, storage technical constraints,} \quad (5b)$$

$$0 \leq \widetilde{p} \leq \overline{p} \quad \forall \xi \in \Xi \quad (5c)$$

Uncertainty set Ξ changes dynamically based on forecasts/ weight samples.



Coefficent matrix for charging



Offer and control actions.

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Summary:

- Integrated forecasting-optimization to improve prescriptive performance in renewable trading applications.
- Explainability of decisions: impact of feature data on decision costs.
- Outperforms the "forecast, then optimize" modeling approach without requiring multiple forecast models.
- Market price forecasting relatively more important for single- than dual-price balancing mechanism.

Future work:

- Moving from batch to online learning setting.
- Interpretable model for data-driven decisions (optimal trees).

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